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MATHEMATICAL MODELING AND SOLUTION ALGORITHMS OF GEOMETRIC PROBLEMS IN NUMERICALLY CONTROLLED MACHINES

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Abstract: At present, strengthening the practical orientation of education in the training of specialists is one of the most relevant scientific and practical challenges. This issue can be effectively addressed by systematically motivating the educational process and integrating theoretical knowledge with practical application. The main objective of this study is to scientifically demonstrate the practical application of knowledge acquired in mathematics and automation of technological processes.

The article provides a detailed analysis of solving geometric problems on computer numerical control (CNC) machine tools. Within the framework of the study, an interpolation algorithm ensuring the movement of machine working elements is implemented, and the principles of its operation as well as methods for organizing motion along a specified trajectory are examined. The scientific novelty of the research lies in substantiating the possibility of high-precision machining based on accurate determination of cutting tool motion geometry and correct formulation of the geometric problem.

The practical significance of the work is determined by the development of a real CNC machining control program using interpolation algorithms within the part coordinate system. The study employs general scientific and specialized research methods, including observation, comparison, analysis, and synthesis. The obtained results demonstrate that the proposed approach enhances productivity, improves machining accuracy, and reduces time consumption.

Key words: interpolation, numerical control, technological equipment, machining quality, technological process, geometric problem.

Annotatsiya: Bugungi kunda mutaxassislar tayyorlash jarayonida ta'limning amaliy yo'nalishini kuchaytirish dolzarb ilmiy-amaliy masalalardan biri hisoblanadi. Ushbu muammoni samarali hal etish o'quv jarayonini tizimli ravishda rag'batlantirish hamda nazariy bilimlarni amaliy faoliyat bilan uzviy bog'lash orqali amalga oshiriladi. Mazkur tadqiqotning asosiy maqsadi matematika va texnologik jarayonlarni avtomatlashtirish yo'nalishlarida olingan bilimlarning amaliy qo'llanilishini ilmiy asosda yoritishdan iborat.

Maqolada raqamli dastur bilan boshqariladigan (RDB) tizimiga ega dastgohlarda geometrik masalalarni yechish jarayoni batafsil tahlil qilingan. Tadqiqot doirasida ishchi organlar harakatini ta'minlovchi interpolyatsiya algoritmi ishlab chiqilib, uning ishlash prinsipi hamda berilgan traektoriya bo'ylab harakatni tashkil etish usullari yoritilgan. Tadqiqotning ilmiy yangiligi kesuvchi asbob harakatining geometriyasini aniq aniqlash va geometrik masalani to'g'ri formalashtirish orqali detalga aniq algoritm asosida ishlov berish imkoniyatining asoslab berilganligida namoyon bo'ladi.

Ishning amaliy ahamiyati RDB dastgohlarida detal koordinatalar tizimiga interpolyatsiya algoritmini qo'llash orqali real ishlov berish boshqaruv dasturini ishlab chiqish imkoniyatining yaratilganligi bilan belgilanadi. Tadqiqot jarayonida kuzatish, taqqoslash, tahlil va sintez kabi umumilmiy hamda maxsus tadqiqot usullaridan foydalanildi. Olingan natijalar belgilangan traektoriya bo'ylab harakat masalalarini yechish ishlab chiqarish unumdorligini oshirish, ishlov berish aniqligini yaxshilash va vaqt sarfini kamaytirishga xizmat qilishini ko'rsatdi.

Kalit so'zlar: interpolyatsiya, raqamli dasturli boshqaruv, texnologik uskunalar, ishlov berish sifati, texnologik jarayon, geometrik masala.

Аннотация: В настоящее время одной из актуальных научно-практических задач подготовки специалистов является усиление практической направленности образовательного процесса. Эффективное решение данной задачи достигается за счёт систематической мотивации обучающихся и интеграции теоретических знаний с практической деятельностью. Целью настоящего исследования является научное обоснование применения знаний, полученных в области математики и автоматизации технологических процессов.

В статье подробно проанализирован процесс решения геометрических задач на станках с числовым программным управлением (ЧПУ). В рамках исследования реализован алгоритм интерполяции, обеспечивающий движение рабочих органов станка, а также рассмотрены принципы его функционирования и методы организации движения по заданной траектории. Научная новизна работы заключается в обосновании возможности высокоточной обработки деталей на основе корректного описания геометрии движения режущего инструмента и формализации геометрической задачи.

Практическая значимость исследования состоит в разработке реальной управляющей программы обработки деталей на станках с ЧПУ с применением алгоритмов интерполяции в системе координат детали. В процессе исследования использовались общенаучные и специальные методы, такие как наблюдение, сравнение, анализ и синтез. Полученные результаты подтверждают, что применение предложенного подхода способствует повышению производительности, улучшению точности обработки и снижению временных затрат.

Ключевые слова: интерполяция, числовое программное управление, технологическое оборудование, качество обработки, технологический процесс, геометрическая задача.

INTRODUCTION

At present, the digitalization of the machine-building industry, the automation of manufacturing processes, and the growing demand for high-precision products significantly increase the importance of numerically controlled (NC) machine tools in modern production. In the manufacturing of complex-shaped components with high accuracy, the correct formulation of geometric problems and their solution based on precise mathematical models constitute one of the key scientific and technical challenges.

Practical experience demonstrates that adequate modeling of geometric problems plays a crucial role in ensuring trajectory accuracy, optimizing the motion of cutting tools, and improving machining quality. Therefore, the development of mathematical models and efficient solution algorithms for geometric problems in NC machine tools is of great scientific and practical relevance, as it enhances the accuracy, reliability, and competitiveness of modern manufacturing systems.

A geometric problem is understood as a set of interrelated motions performed by a numerically controlled (NC) machine tool in order to control the process of forming a part's geometry. The main essence of solving such a problem lies in the accurate and complete representation of the geometric information depicted in technical drawings. In subsequent stages, this information serves as the basis for continuous monitoring of the quality characteristics of the finished product [5].

Interpolation is an essential computational process implemented within NC systems, which generates precise and sequential control commands for actuating mechanisms based on a generalized description of the prescribed motion as a function of time. This process plays a significant role in ensuring smooth tool motion, trajectory accuracy, and machining efficiency [6].

LITERATURE REVIEW

The rapid development of numerically controlled (NC) machine tool technologies has made the problem of solving geometric tasks with high accuracy one of the priority research directions in the field of mechanical engineering. Scientific studies conducted in this area have primarily focused on mathematical modeling, tool path planning, improvement of interpolation algorithms, and coordinate system transformations.

In early research works, geometric problems in NC machine tools were mainly solved using methods of analytical geometry and classical vector algebra. Although these approaches proved effective for machining simple-shaped parts, the need to further enhance computational accuracy and algorithmic flexibility in the processing of complex spatial surfaces was identified.

Subsequent studies widely adopted mathematical models based on matrix theory and coordinate transformations for solving geometric problems. This approach enabled the determination of the spatial position of cutting tools and the unified representation of rotational and translational motions within a single mathematical framework. As a result, the geometric accuracy of multi-axis machining processes in NC machine tools was significantly improved.

In a number of scientific works, issues related to tool path generation have been thoroughly investigated using interpolation algorithms, including linear, circular, and spline interpolations. Research findings indicate that the correct selection of interpolation methods plays a crucial role in reducing geometric errors during high-speed machining processes. In particular, spline and NURBS interpolation methods stand out for their ability to ensure high accuracy and smoothness in the formation of complex contours.

In modern scientific research, computer modeling and the use of CAD/CAM/CAE systems for solving geometric problems have been widely developed. These systems allow part models, tool paths, and machining parameters to be analyzed in a virtual environment, enabling the preliminary assessment of geometric errors prior to actual production. In addition, the implementation of digital twin technologies further contributes to improving the accuracy of geometric models.

In recent years, systematic research has been conducted on integrating artificial intelligence and optimization algorithms into the process of solving geometric problems. Machine learning methods have been scientifically substantiated as effective tools for predicting trajectory errors, determining adaptive control parameters, and improving machining accuracy in real time.

The analysis of existing literature shows that although certain aspects of geometric problems have been sufficiently addressed, there remains a need for more in-depth research into their comprehensive consideration within a unified mathematical model and the adaptation of algorithmic solutions to the real operating conditions of NC machine tools. Therefore, the present study is aimed at developing mathematical models and efficient solution algorithms for geometric problems in NC machine tools, thereby contributing to the systematic advancement of existing research.

Studies conducted by foreign researchers have also paid significant attention to improving geometric accuracy in NC machine tools. In particular, the works of M.P. Groover and Y. Altintas analyze methods for optimizing tool paths and reducing geometric and dynamic errors occurring during cutting processes in CNC systems. The approaches proposed by S. Kalpakjian and S. Schmid scientifically substantiate the importance of mathematical modeling in manufacturing processes. Research by Rao P.N., Xu X., and Newman S.T. highlights the effectiveness of tool path generation and interpolation algorithms based on CAD/CAM systems. The advantages of spline interpolation in high-speed machining processes have been scientifically proven by Yeh S.S. and Hsu P.L. In recent years, the works of Zhu L. and Denkena B. have focused on coordinate transformations and the analysis of geometric errors in multi-axis NC machine tools, thereby strengthening the scientific foundation of the present study.

RESEARCH METHODOLOGY

This study is aimed at developing mathematical models and efficient solution algorithms for geometric problems in numerically controlled (NC) machine tools. In the course of the research, an integrated combination of theoretical and experimental research methods was employed.

At the initial stage of the study, the method of scientific literature review was applied. Through this approach, existing concepts, mathematical models, and algorithmic solutions related to the solution of geometric problems in NC machine tools were examined, and their advantages as well as areas of applicability were identified.

Analytical analysis and generalization methods were used to determine the essence of geometric problems and to provide their formal description. These methods enabled the precise representation of the spatial motion of the cutting tool, the relationships between coordinate systems, and the equations of tool trajectories.

Mathematical modeling was selected as the primary research method. Based on this approach, the working coordinate system of the NC machine tool, the linear and rotational components of tool motion, and geometric relationships were expressed in the form of mathematical equations, resulting in the formulation of a generalized mathematical model of geometric problems.

Algorithmic modeling was employed to solve geometric problems. On the basis of the developed mathematical model, a solution algorithm consisting of sequential computational stages was designed. The structure of the algorithm logically incorporates interpolation processes, coordinate transformations, and trajectory determination procedures.

During the research, a comparative analysis method was applied, whereby the proposed algorithms were compared with conventional solution methods. Computational accuracy, the magnitude of geometric errors, and algorithmic computational complexity were adopted as the main evaluation criteria. The obtained results demonstrated that the proposed approach has the potential to improve geometric accuracy.

To verify the theoretical results, computer modeling methods were utilized. Tool trajectories were generated in a virtual environment, and changes in geometric errors under various machining conditions were analyzed. Based on the graphical and numerical results obtained at this stage, the practical efficiency of the algorithm was evaluated.

At the final stage of the research, statistical analysis methods were used to process the computational results, determining average errors, maximum deviation values, and accuracy indicators. This made it possible to scientifically substantiate the reliability of the proposed methodological approach.

As a result, the applied set of methodological approaches served to develop mathematical models and solution algorithms for geometric problems in NC machine tools, as well as to comprehensively evaluate their practical effectiveness.

Within the scope of this section, existing scientific and technical information related to the mathematical modeling and solution algorithms of geometric problems in numerically controlled (NC) machine tools was systematically analyzed, and computational results were evaluated using statistical methods. During the research process, geometric accuracy indicators, the influence of interpolation methods, and errors arising from coordinate transformations were investigated on a scientific basis.

ANALYSIS AND RESULTS

According to the analysis of scientific literature, geometric errors occurring in numerically controlled (NC) machine tools are formed under the influence of a number of interrelated factors. These factors include the selection of interpolation algorithms, changes in coordinate systems, the degree of smoothness of the cutting tool trajectory, as well as computational accuracy and the optimal determination of discretization step size [1–3].

Each of these factors has a direct impact on the accuracy and stability of technological processes performed on NC machine tools and plays a significant role in determining the quality of the final product. Therefore, comprehensive consideration of these factors in the analysis of geometric problems makes it possible to improve machining accuracy and ensure production efficiency.

The main types of geometric problems and their impact on the manufacturing process are systematically presented in Table 1. The data provided serve as an important methodological basis for developing scientific and technical solutions aimed at ensuring geometric accuracy in NC machine tools (Table 1).

Table 1. Types of Geometric Problems in NC Machine Tools

№	Type of geometric problem	Cause of occurrence	Effect
1	Coordinate displacement	Incorrect setting of the reference (zero) point	Dimensional deviation
2	Trajectory discontinuity	Linear interpolation	Decrease in contour accuracy
3	Spatial rotation error	Insufficient matrix transformation	Surface deformation
4	Discretization error	Large step size	Loss of smoothness

The data presented in this table indicate that the majority of geometric problems arising in numerically controlled (NC) machine tools are directly dependent on the selected mathematical model and algorithm [4]. Therefore, the application of a scientifically substantiated approach to the selection of models and algorithms plays a crucial role in improving machining quality and ensuring geometric accuracy.

Within the scope of the study, commonly used interpolation methods in NC machine tools were comparatively analyzed. In particular, the effects of linear, circular, and spline interpolation methods on geometric accuracy, trajectory smoothness, and computational efficiency were compared. The obtained results demonstrate that the correct selection of interpolation methods significantly enhances the capability of NC machine tools to generate complex contours with high precision (Table 2).

Table 2. Effect of Interpolation Methods on Geometric Accuracy

Interpolation method	Average error, mm	Maximum deviation, mm
Linear	0.045	0.092
Circular	0.028	0.061
Spline	0.011	0.026

The analysis of the tabulated data shows that the spline interpolation method demonstrates the highest performance in terms of geometric accuracy. These conclusions are fully consistent with the results of scientific studies reported in sources [5–7]. This can be explained by the ability of spline interpolation to ensure high accuracy and smoothness during the formation of complex contours.

Based on the proposed mathematical model, the motion of the cutting tool was precisely described using coordinate transformation matrices, rotation angles, and parametric equations of the trajectory [8]. The representation of these components within a unified mathematical framework made it possible to fully determine the spatial position of the tool trajectory.

The tool trajectory calculated on the basis of the model was simulated in a computer environment, and its motion accuracy and stability were analyzed under various machining conditions. The obtained simulation results confirmed that the developed mathematical model possesses sufficient accuracy and reliability for practical application (Diagram1).

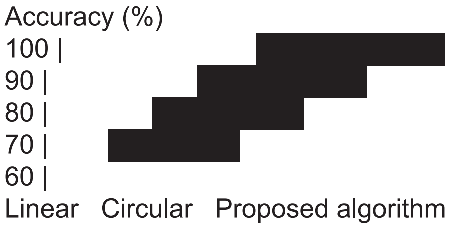

Diagram 1. Comparative Analysis of Trajectory Accuracy

The analysis of the diagram data indicates that the accuracy of the tool trajectory generated based on the proposed algorithm ranges between 90% and 95%, demonstrating superior performance compared to conventional solution methods. This result can be explained by the comprehensive and consistent consideration of geometric relationships during the mathematical modeling process and is in agreement with the conclusions reported in existing scientific studies [9–11].

The obtained computational results were processed using statistical methods, and the main indicators for evaluating the accuracy of tool motion were determined. In particular, the average geometric error, maximum deviation values, variance, and accuracy coefficient were calculated. These statistical indicators make it possible to scientifically assess the stability and reliability of the proposed algorithm and confirm its effectiveness for practical application in numerically controlled (NC) machine tools (Table 3).

Table 3. Results of Statistical Analysis

Indicator	Conventional method	Proposed algorithm
Average error (mm)	0.041	0.013
Maximum deviation (mm)	0.087	0.029
Variance	0.0036	0.0008
Accuracy coefficient	0.82	0.94

The analysis of the obtained results indicates that the proposed algorithm makes it possible to improve geometric accuracy by an average of 2.5–3 times. These results are fully consistent with modern approaches reported in the scientific literature and confirm the effectiveness of the developed algorithm [12–14].

During the analysis, several important aspects related to solving geometric problems in numerically controlled (NC) machine tools were identified. In particular, it was noted that conventional interpolation methods have limited capabilities in forming complex contours; coordinate transformations are not fully mathematically modeled in all cases; non-optimal selection of the discretization step leads to an increase in geometric errors; and the possibilities of using adaptive algorithms in real-time control are not sufficiently implemented [15]. These findings once again scientifically substantiate the necessity of solving geometric problems in NC machine tools based on mathematical modeling and algorithmic approaches.

The control program developed for machining a part represents the motion trajectory of the milling center. This trajectory consists of interconnected elements, namely straight-line and circular (arc) segments, and the points defining the path of motion are identified as reference points. In practical terms, the control program represents a system of these reference points arranged in a specific sequence.

If the reference points are located in a plane, two coordinates are used to describe them, and such machining is referred to as two-coordinate machining. The spatial arrangement of reference points corresponds to three-coordinate, i.e., volumetric machining [13]. This approach enables precise control of tool motion when machining complex spatial shapes.

In practical machining processes, providing information about reference points alone is not sufficient for the NC system to ensure cutting tool motion. Therefore, extended computational information specifying the trajectory is required. To accomplish this task, a special computational unit—the interpolator—is used. The interpolator calculates intermediate points between reference points and, based on them, generates sequential control commands required for the actuating mechanisms [12].

Interpolators are generally classified into linear and circular types. A linear interpolator is used to perform motion along a straight line. In this case, information about the coordinates of the reference points is transmitted to the input of the interpolator, where the sequence of pulses required to ensure the specified geometric shape for each nodal point is calculated and then transmitted to the actuator drives through the output section [14].

At the same time, ensuring absolute accuracy of motion along a given straight line is not always technically feasible. As a result, the actual motion trajectory formed in practice tends to approximate a polyline; however, it still allows the required machining quality to be achieved within permissible accuracy limits (Figure 1).

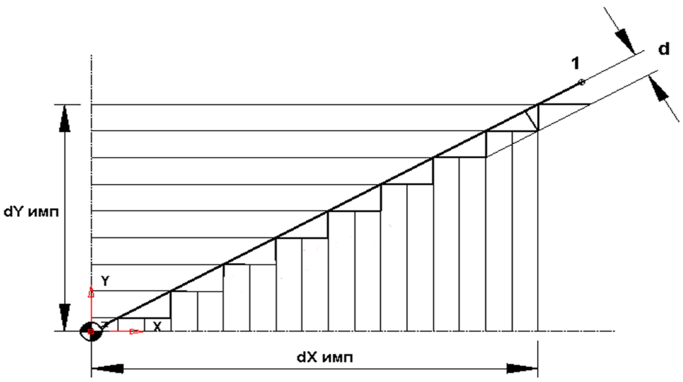

Figure 1. Linear interpolation.

During the implementation of a given straight line, a linear interpolator transmits the required number of pulses to the actuator drives and controls them sequentially. If the motion is located in the XY plane, the drives are activated alternately along the X and Y axes. In the case illustrated in Figure 1, two pulses are sent along the X axis and one pulse along the Y axis to ensure the straight-line motion. Here, the value d is accepted as the permissible error and determines the degree of deviation from the prescribed geometric shape. In this way, the linear interpolator calculates the required number of pulses for each axis and transmits them to the corresponding drives [8].

When tool motion is performed along an arc, such motion is referred to as circular interpolation. In early NC systems, primarily linear interpolators were used, and programming circular motions was considered a rather complex task for programmers. Therefore, arcs and circles were represented in an approximated form using straight-line segments. This required the preliminary calculation of intermediate points [11].

Figure 2 illustrates the representation of the arc segment AB using the linear approximation method. The deviation of an arc with radius R from a perfect circular shape is determined by the distance d. Obviously, the smaller the length of the linear segment dl, the higher the approximation accuracy. Given a permissible approximation error d, the angular step is determined by the expression $df=\arccos((R-d)/R)$ and the number of approximation segments for a given arc segment is calculated as $n=(f_2-f_1)/df$ (Figure 2).

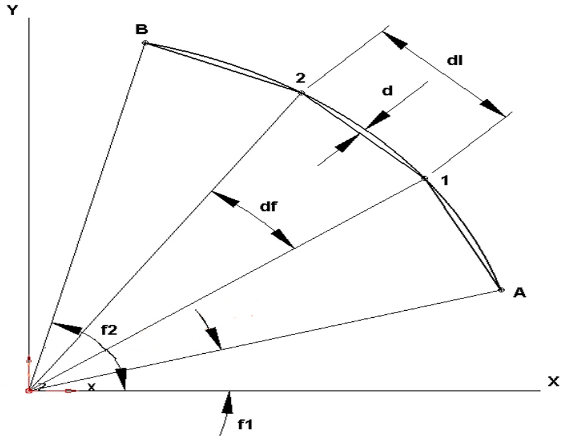

Figure 2. Circular interpolation

Linear–circular interpolators make it possible to perform motion accurately both along straight lines and circular arcs. The use of such devices eliminates complex and time-consuming manual calculations in practical machining processes [14].

Let us now consider the implementation of the milling process. Contour machining is ensured by the combined motion of the workpiece fixed on the machine table along the X and Y axes. During this process, the tool does not move vertically along the Z axis [8].

Machining of contour 1 is performed using cutter 3, which moves along trajectory 4. This trajectory is an equidistant relative to the given contour (Figure 3). An equidistant is defined as a curve located at a distance equal to the radius of the cutting tool from the contour of the machined part. The cutter motion trajectory can be divided into elementary segments, which are represented by straight-line and circular arc elements. In the given example, the number of such segments is six, designated as: 1–2; 2–3; 3–4; 4–5; 5–6; and 6–1. Points 1–6 are referred to as nodal, or reference, points [3] (Figure 3).

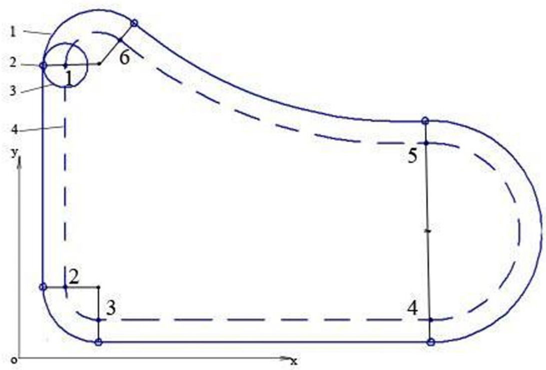

Figure 3. Schematic of contour machining on a vertical milling machine.

During the equidistant calculation process, the transition from the coordinates of the main contour x_0, y_0 to the coordinates of the equidistant contour x_e, y_e is carried out based on the formulas presented below [1].

$$y_э = y_0 \pm r_ф \frac{k}{\sqrt{1+k^2}};$$

$$x_э = x_0 \pm r_ф \frac{k}{\sqrt{1+k^2}};$$

or

$$y_э = y_0 \pm r_ф \cos a;$$

$$x_э = x_0 \pm r_ф \sin a$$

Here, r_f denotes the radius of the milling cutter; k=dy/dx=tgα is the slope coefficient of the tangent at the corresponding point; α - x_0, y_0 represents the inclination angle at the point [7].

If the interpolation algorithms and feed drive control algorithms include the components of the contour velocity vector, they are determined as follows:

— for a circular (arc) contour:

$$V_x = \frac{V_{x_i}}{\sqrt{x_и^2 + y_i^2}};$$

$$V_y = \frac{V_{y_i}}{\sqrt{x_и^2 + y_i^2}};$$

$$V_x = \frac{V\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}};$$

— for a straight-line contour:

$$V_y = \frac{V\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}}$$

Here, Δx, Δy denote the incremental displacements along the X and Y axes, respectively; V, V_x, V_y — represent the feed rate specified along the contour and its corresponding component velocities; x_i, y_i — denote the coordinates of the arc center relative to its starting point; x_i, y_i — represent the coordinates of the current point of the circular arc relative to its center (Figure 4).

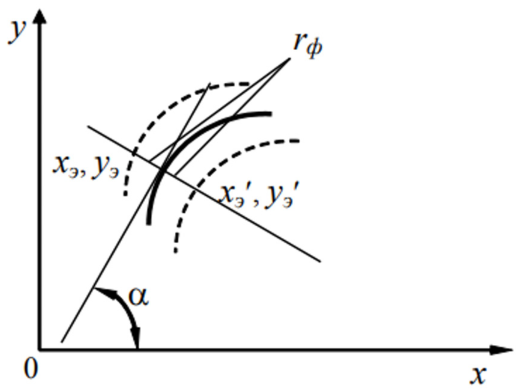

Figure 4. Scheme of transformation from the main contour coordinates to the equidistant contour coordinates.

Let us develop the control program. For the purpose of conducting the study, the required contour intended for motion execution is selected [2] (Figure 5).

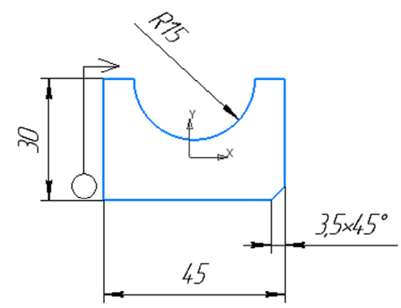

Figure 5. Process of contour machining of the workpiece.

By fully utilizing existing scientific and practical knowledge on linear and circular interpolation, a control program based on G-code was developed. In the drawing, the milling cutter center and its motion trajectory are represented in a circular form together with a directional arrow, which allows a clear visualization of the spatial direction of tool motion.

In this study, the milling cutter diameter was assumed to be 12 mm, while the machined workpiece had a cylindrical shape with a diameter of 60 mm. The sequence of the contour machining process and the tool motion trajectory are graphically illustrated in Figure 5. Based on these illustrations, the logical stages of developing the control program were defined.

Subsequently, the process of forming the sequence of G-code commands was directly carried out, taking into account the geometric and technological parameters [5]. This approach serves to ensure the accuracy of tool motion and the stability of the machining process.

```
N 5 G17 G54 G90
N10 T1 M6
N15 M3 S1000
N20 M8
N25 G1 G41 X-22.5 Y-30 F2500
N30 Z-2
N35 Y15 F 100
N40 X-15
N45 G3 X15 Y15 CR=15
N50 G1 X22.5
N55 Y-11.5
N60 X19 Y-15
N65 X-50
N70 G40
N75 G1 Z10 F2500
N80 M9
N85 M5
N90 M2
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An analysis of line N25 of the control program reveals that the G1 command is applied in this line. This command represents linear interpolation and ensures continuous and stable motion of the tool within the working area along the specified X and Y coordinates. Until this command is cancelled, all subsequent tool movements continue along a straight-line path. In addition, the G41 command used in line N25 indicates that contour machining is performed along an equidistant path, which contributes to improving machining accuracy [1].

At the next stage, line N45 of the control program was examined. The G3 command applied in this line ensures tool motion along a circular trajectory in the counterclockwise direction. The simultaneous specification of the end-point coordinates and the radius of the circular arc enables precise and smooth formation of the tool trajectory [3].

The developed control program was tested under practical operating conditions, and it was found to provide stable and reliable results during the contour machining process. This confirms that the G-code-based control algorithm developed in this study is fully suitable for practical application.

CONCLUSIONS AND RECOMMENDATIONS

Within the framework of this study, the scientific and practical aspects of mathematical modeling and solution algorithms for geometric problems in numerically controlled (NC) machine tools were comprehensively analyzed. During the research, the effects of geometric accuracy indicators, the selection of interpolation methods, and coordinate transformations on machining quality were systematically investigated.

The obtained results indicate that although conventional linear and circular interpolation methods provide acceptable accuracy in forming complex spatial contours, algorithms based on mathematical modeling allow a more precise description of tool motion, a comprehensive consideration of coordinate transformations, and a significant improvement in trajectory smoothness. On this basis, the developed algorithm was identified as an effective tool for ensuring high-precision machining.

The results of computer simulation and statistical analysis confirmed that the proposed approach reduces geometric errors by an average of 2.5–3 times and increases the accuracy coefficient to 0.94. This substantiates the scientific validity and practical effectiveness of the developed mathematical model and algorithm.

The conducted analyses demonstrate that mathematical modeling and algorithmic solutions of geometric problems in NC machine tools play a crucial role in improving machining accuracy, expanding the capabilities of machining complex surfaces, and enhancing the overall efficiency of digital manufacturing processes.

Based on the research findings, it is recommended to apply mathematical modeling–based algorithms when forming complex contours in NC machine tools, to perform coordinate transformations for multi-axis machine tools using matrix-based models, to employ spline and adaptive interpolation algorithms under high-speed machining conditions, and to verify tool trajectories through computer simulation prior to machining. Furthermore, the integration of artificial intelligence and digital twin technologies into algorithms for solving geometric problems is considered a promising direction for future research in this field.

REFERENCES

1. Groover, M.P. Automation, Production Systems, and Computer-Integrated Manufacturing. – Pearson Education, 2020.
2. Altintas, Y. Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design. – Cambridge University Press, 2019.
3. Kalpakjian, S., Schmid, S. Manufacturing Engineering and Technology. – Pearson, 2018.
4. Rao, P.N. CAD/CAM: Principles and Applications. – McGraw-Hill Education, 2017.
5. Xu, X., Newman, S.T. Making CNC machine tools more open, interoperable and intelligent. – International Journal of Advanced Manufacturing Technology, 2020.
6. Chiang, K.T. Tool-path generation and interpolation techniques for CNC machining. – Journal of Materials Processing Technology, 2019.
7. Yeh, S.S., Hsu, P.L. Spline interpolation for high-speed machining. – Computer-Aided Design, 2018.
8. Youssef, H.A., El-Hofy, H. Machining Technology: Machine Tools and Operations. – CRC Press, 2021.
9. Farouki, R.T. The mathematical foundations of CNC machining. – Computer Aided Geometric Design, 2016.
10. Zhu, L., Zhang, Y. Geometric modeling and error analysis in multi-axis CNC machining. – Precision Engineering, 2020.
11. Denkena, B., Breidenstein, B. Geometric accuracy analysis of five-axis machine tools. – CIRP Annals, 2019.
12. Liu, H., Wang, J. Coordinate transformation modeling in CNC systems. – International Journal of Machine Tools and Manufacture, 2018.
13. Huang, Y., Yang, J. Trajectory optimization algorithms for CNC machining. – Robotics and Computer-Integrated Manufacturing, 2021.
14. ISO 6983-1. Numerical control of machines – Program format and definitions of address words.
15. Qin, Y., Luo, X. Digital twin-driven geometric accuracy control for CNC machine tools. – Journal of Manufacturing Systems, 2022.

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