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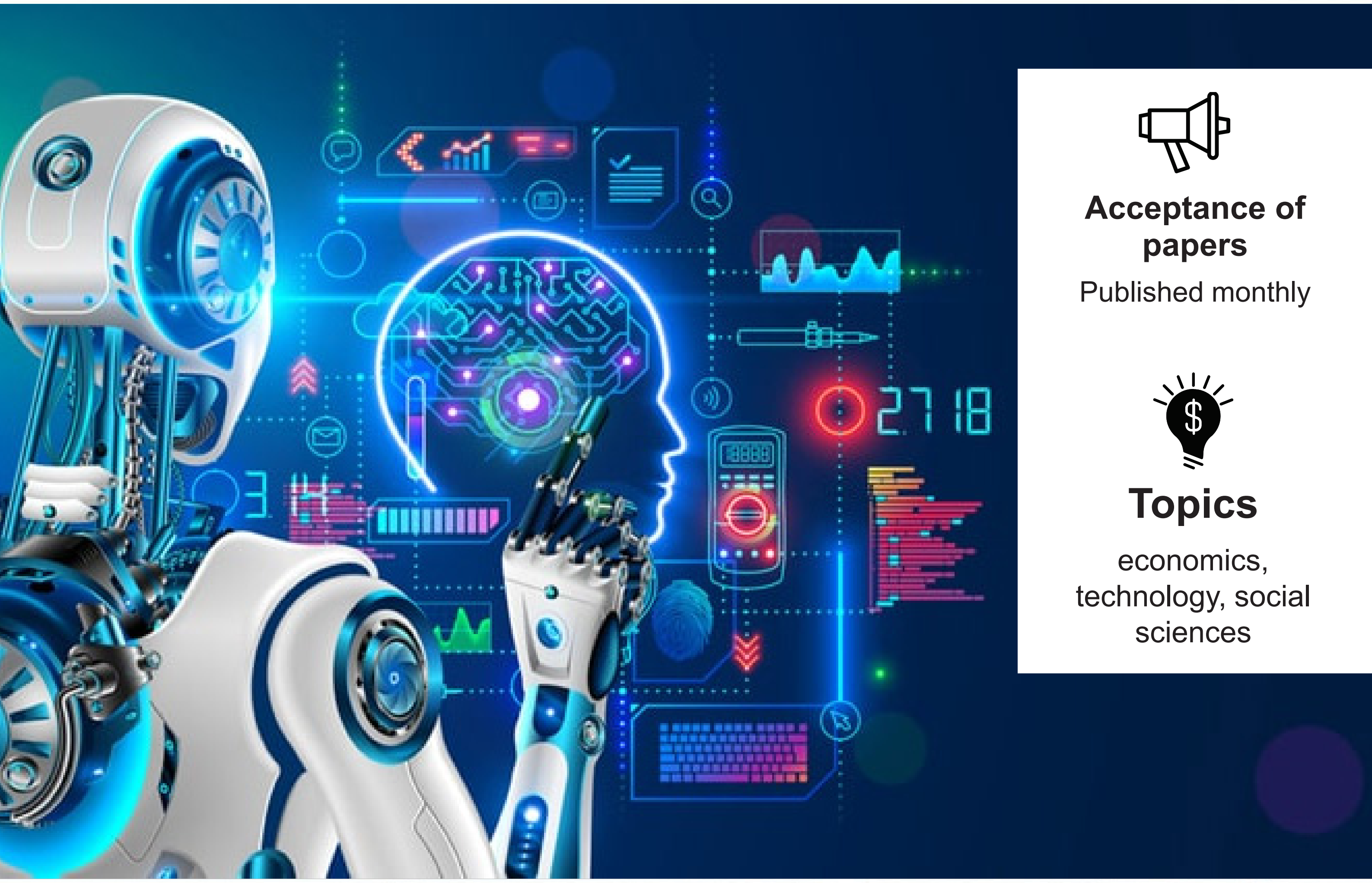

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CONTENTS

PROPAGATION OF SMALL MOTIONS IN A TWO-LAYER DISPERSE MEDIA FLOW

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Abstract: In this article, the propagation of small perturbations occurring in the flow of a two-layer dispersed medium is studied. Two-layer dispersed flows are common in many natural and technological processes, including oil and gas pipelines, chemical reactors, and biological environments. The appearance and development of small shocks in such systems seriously affects the stability of the flow. In the study, a mathematical model was built around the main flow state using the small parameters method and the differential equations were linearized. Differences in density, velocity, and concentration of dispersed phases between the layers were analyzed in terms of their influence on the propagation speed and stability of the excitations. Dispersion relations were obtained and laws of development of movements in time and space were studied. The obtained results show that excitations may be suppressed in some parameter values, while in other cases they may increase. The results of the research are of practical importance in modeling two-layer dispersed flows and evaluating their stability.

Key words: hydrodynamics, pressure pulse, dispersed mixture, Laplace's equation, velocity equation, velocity potential.

Annotatsiya: Mazkur maqolada ikki qatlamli dispers muhit oqimida yuzaga keladigan kichik qo‘zg‘alishlarning tarqalishi tadqiq etilgan. Ikki qatlamli dispers oqimlar ko‘plab tabiiy va texnologik jarayonlarda, jumladan, neft-gaz quvurlari, kimyoviy reaktorlar hamda biologik muhitlarda keng uchraydi. Bunday tizimlarda kichik qo‘zg‘alishlarning paydo bo‘lishi va rivojlanishi oqim barqarorligiga jiddiy ta’sir ko‘rsatadi. Tadqiqotda asosiy oqim holati atrofida kichik parametrlar usuli yordamida matematik model qurilgan va differensial tenglamalar chiziqlashtirilgan. Qatlamlar orasidagi zichlik, tezlik va dispers fazalar konsentratsiyasi farqlari qo‘zg‘alishlarning tarqalish tezligi hamda barqarorligiga ta’siri nuqtayi nazaridan tahlil qilingan. Dispersiya munosabatlari olingan va qo‘zg‘alishlarning vaqt hamda fazo bo‘yicha rivojlanish qonuniyatlari o‘rganilgan. Olingan natijalar ayrim parametr qiymatlarida qo‘zg‘alishlar so‘nishi, boshqa holatlarda esa ularning kuchayishi mumkinligini ko‘rsatadi. Tadqiqot natijalari ikki qatlamli dispers oqimlarni modellashtirish va ularning barqarorligini baholashda amaliy ahamiyatga ega.

Kalit so‘zlar: gidrodinamika, bosim impulsi, dispers aralashma, Laplas tenglamasi, tezlik tenglamasi, tezlik potensiali.

Аннотация: В данной статье исследуется распространение малых возмущений, возникающих в потоке двухслойной дисперсной среды. Двухслойные дисперсные потоки распространены во многих природных и технологических процессах, включая нефте- и газопроводы, химические реакторы и биологические среды. Возникновение и развитие малых ударных волн в таких системах серьезно влияет на устойчивость потока. В исследовании была построена математическая модель основного состояния потока с использованием метода малых параметров, а дифференциальные уравнения были линеаризованы. Различия в плотности, скорости и концентрации дисперсных фаз между слоями были проанализированы с точки зрения их влияния на скорость распространения и устойчивость возбуждений. Были получены дисперсионные соотношения и изучены законы развития движений во времени и пространстве. Полученные результаты показывают, что возбуждения могут подавляться при некоторых значениях параметров, в то время как в других случаях они могут усиливаться. Результаты исследования имеют практическое значение при моделировании двухслойных дисперсных потоков и оценке их устойчивости.

Ключевые слова: гидродинамика, импульс давления, дисперсная смесь, уравнение Лапласа, уравнение скорости, потенциал скорости.

INTRODUCTION

The stability of flows in two-layer dispersed media and the small perturbations arising within them constitute one of the important directions of modern hydrodynamics and multiphase flow research. Two-phase flows occur in numerous scientific and engineering processes, such as phase interactions in oil and gas pipelines, liquid–

solid flows in reactors, and microfluidic devices, which necessitates the study of their stability characteristics. For example, in analyzing the stability of two-phase flows, the effects of variable density and velocity at the interface are considered key factors, as they determine the mechanism of perturbation propagation [1].

It is well known that small disturbances arising between layers can alter the stability of the interface in the flow structure. These processes, under the influence of pressure, velocity, and thermal conditions, give rise to various types of instabilities, including well-studied static thermal instabilities [2]. At the same time, small perturbations are also of great importance in analyzing the stability of interfaces between laminar and turbulent flows, since they can lead to changes in intensity and to exchange processes between phases.

In recent studies, new numerical methods and models have been proposed to analyze instabilities in two-phase flows, including theoretical investigations of the stability of two-phase flows in parallel channels and analyses of the influence of industrial parameters [3], as well as the development of modern finite element methods for modeling small disturbances in turbulent multiphase flows [4]. These studies provide an important theoretical and practical foundation for investigating the propagation of interfacial perturbations and the conditions of stability in two-layer dispersed flows.

In this article, the propagation characteristics of small perturbations in a two-layer dispersed medium flow are analyzed through mathematical modeling. As the main problem, linearized equations and dispersion relations are considered, which provide a fundamental basis for assessing flow stability.

REVIEW OF LITERATURE ON THE SUBJECT

The study of wave and disturbance propagation in multiphase and layered media has a long and well-established tradition in fluid mechanics, continuum mechanics, and applied mathematics. Early theoretical foundations of small-amplitude motion in fluids were laid within classical hydrodynamics, where linearization techniques allowed researchers to describe wave propagation under weak perturbations. In this context, Horace Lamb's Hydrodynamics published in 1932 remains a cornerstone, providing rigorous formulations for small oscillations in stratified and layered fluids, which later became essential for analyzing multi-layer flow systems.

The development of dispersion theory in fluid flows significantly expanded the understanding of how wave speed depends on frequency and medium structure. Lev Landau and Evgeny Lifshitz, in their seminal work Fluid Mechanics released in 1959, systematically analyzed linear waves in continuous media, emphasizing the role of compressibility, density stratification, and interfacial effects. Their theoretical framework has been widely applied to layered and multiphase flows, where dispersion relations govern the stability and evolution of small perturbations.

Research on two-layer flows gained particular momentum in the second half of the twentieth century due to applications in geophysical and industrial systems. Joseph Boussinesq's earlier ideas on stratified fluids were later refined by modern scholars such as Lawrence S. Leibovich, who in 1983 examined wave–mean flow interactions in layered fluids, highlighting how weak disturbances can exchange energy between layers. These studies demonstrated that even small motions in layered systems may lead to complex dynamic behavior due to interfacial coupling.

Disperse media flows, involving suspended particles, bubbles, or droplets, introduce additional physical mechanisms that alter wave propagation. In this area, Daniel Drew and Stephen Passman made a major contribution with their book Theory of Multicomponent Fluids published in 1999. They developed a rigorous continuum approach for multiphase mixtures, showing how relative motion between phases affects momentum transfer and dispersion characteristics. Their work provides a theoretical basis for analyzing small disturbances in two-layer disperse flows, where each layer may possess different phase compositions.

Another important direction concerns the stability and wave dynamics of stratified multiphase flows. In 2005, Andrea Prosperetti and Grétar Tryggvason presented a comprehensive overview of computational and theoretical methods for multiphase flow analysis, emphasizing the role of interfacial forces and phase interactions in wave propagation. Their findings underline that dispersion relations in such systems are strongly influenced by particle inertia, volume fraction, and interphase drag, all of which are critical in modeling small motions.

From a mathematical perspective, the propagation of small perturbations in layered disperse media has been studied using asymptotic methods and linear stability analysis. Gérard Iooss and Daniel D. Joseph, in their 1990 work on elementary stability and bifurcation theory, provided tools that are frequently used to analyze the onset and evolution of wave modes in stratified flows. These methods allow researchers to identify conditions under which small disturbances either decay or grow, potentially leading to nonlinear regimes.

More recent studies have focused on practical applications, particularly in environmental and engineering systems. In 2010, C. S. Yih's investigations into stratified flow instability demonstrated how viscosity contrasts between layers significantly modify wave propagation behavior. Similarly, research by Brian J. Glasser and colleagues in 2012 on particulate flows highlighted that even dilute dispersions can substantially alter dispersion relations due to particle–fluid interactions.

Overall, the existing literature demonstrates that the propagation of small motions in two-layer disperse media is governed by a complex interplay of dispersion, interfacial dynamics, and phase interactions. Classical hydrodynamic theories provide the foundation, while modern multiphase flow models extend these concepts to account for dispersed phases. Despite significant progress, further research remains necessary to integrate hydrodynamic theory with realistic disperse media properties, particularly for short-time impulse propagation and transient wave phenomena in layered systems.

RESEARCH METHODOLOGY

The study employs analytical and numerical methods based on linearized hydrodynamic equations for two-layer disperse media. Input data are obtained from predefined physical parameters of each layer, including density, dispersion characteristics, and interphase interaction coefficients. The analysis is conducted through dispersion relations, stability criteria, and short-time impulse response evaluation using mathematical modeling and comparative simulation results.

Analysis and results

Many problems of hydrodynamics are devoted to the study of wave-like motions of single-phase and multiphase fluids and to the application of these motions to situations encountered in hydraulic engineering.

It is known that when a wave propagates over a short time interval $(t \ll 1)$, it generates a large pressure, and the duration of the pressure impulse is short $(0 < t \leq \tau)$. However, although the pressure created over this time interval has a finite impulse, the velocity of the particles set in motion does not become extremely large and remains finite. The pressure impulse of the mixture velocity potential P_m can be determined using the corresponding relation, and its phase-related components are expressed as follows:

$$p_{\tilde{n}\grave{i}}^{(m)} = \left(f_{1\tilde{n}\grave{i}}^{(m)} \rho_{1i} + f_{2\tilde{n}\grave{i}}^{(m)} \rho_{2i} \right) \varphi_{\tilde{n}\grave{i}}^{(m)},$$

where m is the layer index $(m = 1,2)$; $\rho_m^{(n)}$ is the density of the n-th phase in the m-th layer; $f_m^{(n)}$ denotes the true density and volumetric concentration of the n-th phase in the m-th layer.

For each of the two layers of the water basin $(m = 1,2)$ and for each of the two phases, the velocity vectors are denoted by $\vec{v}_m^{(n)}(x, y, z, t)$. We assume that the corresponding velocity potentials $\varphi_m^{(n)}(x, y, z, t)$ exist. Then the velocity vectors are determined through the velocity potentials as:

$$\vec{V}_n^{(m)}(x, y, z, t) = grad\,\varphi_n^{(m)}(x, y, z, t).$$

Using the formula for the mixture velocity potential, it is possible to determine the velocity potential for each phase and, on this basis, the corresponding velocity vectors. Furthermore, under the action of the pressure impulse P_m, it becomes possible to determine the laws of motion of water-saturated soil particles on the surface of the Earth's crust and to identify the position of each particle at a given point in time within the mixture. These data provide the necessary results for evaluating the deformation state of each layer of the Earth's crust.

This approach represents a hydrodynamic method for studying seismic vibrations of the Earth and can be applied to determine the coordinates of the earthquake source and other required parameters [5]. It is known that the distribution of velocities in the Earth's crust is volumetric and follows time-dependent laws. Based on the velocity distributions obtained from the formula of L. I. Sedov [6], it becomes possible to determine the external impulse of seismic vibrations in the Earth's crust. Therefore, the hydrodynamic method presented here enables the identification of natural geophysical fields.

The stress state at points of the Earth's crust is described by stress tensors p_{ij} and velocity vectors v_j, which makes it possible to determine the stress state. These parameters characterize the mechanical state of the crust resulting from seismic processes. Consequently, the interaction of multiphase mixtures between layers and the influence of phases on each other play an important role in the dynamics of dispersed mixtures. In dispersed mixtures, the motion of phases differs from that of ideal fluids.

For a single-phase incompressible fluid, the existence of a velocity potential under the action of impulse forces is theoretically justified [7]. However, in nature, fluids are not ideal; in particular, dispersed mixtures are viscous fluids. Therefore, the hydrodynamic interpretation of velocities in dispersed mixtures is approximated by analogy with the potential velocity interpretation of two-phase fluids. Under this assumption, changes in the concentration of each phase in the mixture are considered negligible.

Therefore, let a large pressure p^* act on a finite volume of an incompressible fluid over a short time interval $0 < t\,(\ll \infty)$. It is defined by the following equation:

$$p_\tau = \lim_{\tau \to 0}\left(\int_0^\tau p^* dt\right) < \infty \quad . \qquad (1)$$

In this case, taking into account the interphase interaction and mutual penetration occurring in an n-phase dispersed mixture flow, the equation describing the motion of the n-th phase of the mixture can be written in the following form:

$$\frac{d\vec{V}_n}{dt} = \vec{F}_n - \frac{1}{\rho_{ni}} gradP + K(\vec{V}_p - \vec{V}_n)_{(p \neq n)} \quad . \qquad (2)$$

By multiplying the equation of motion by the short time interval dt and integrating it over the finite interval $(0, \tau)$, we obtain the following equation:

$$\int_0^\tau \frac{d\vec{V}_n}{dt} dt = \int_0^\tau d\vec{V}_n = \int_0^\tau \vec{F}_n dt - \int_0^\tau \frac{1}{\rho_{ni}} gradP d\tau + \int_0^\tau K(\vec{V}_p - \vec{V}_n) dt \quad . \qquad (3)$$

In order to determine the law of variation of mutually penetrating and interacting mixture phases, we pass to the limit $\tau \to 0$ in equation (3) [8]. Taking into account the boundedness of the mass forces and interphase interaction forces, as well as the fact that the limit of the coupling terms is equal to zero, we obtain from equation (3) the following limiting relation:

$$\vec{V}'_n - \vec{V}_n(0) = -\lim_{\tau \to 0}\int_0^\tau \frac{1}{\rho_{ni}} gradPdt \quad . \qquad (4)$$

If the dispersed mixture under consideration is compressible, $\rho(p) = \rho_n \neq \text{const}$, and the process is assumed to be barotropic, $p = p(\rho)$, then equation (4) takes the following form:

$$\vec{V}_n(\tau) - \vec{V}_n(0) = -\lim_{\tau \to 0}\int_0^\tau gradP_n dt \quad . \qquad (5)$$

Here, P_n is a pressure function defined as:

$$dP_n = \frac{dp}{\rho_{in}} \quad .$$

Here, $\vec{V}_n(0)$ and $\vec{V}_n(\tau)$ are the velocities of the particles of the n-th phase, representing the initial and final velocities under the action of the pressure impulse, respectively.

Passing to the limit in equation (5), we find that, over a short time interval, the action of the pressure impulse does not change the coordinates of the phase particles:

$$\vec{V}_n(\tau) - \vec{V}_n(0) = gradP_n = grad\varphi_n \, .$$

Using this result, we see that the effect of an instantaneous pressure impulse is equal to the increment of the velocity potential of the n-th phase multiplied by the reduced density of the mixture, and that it is directed opposite to the flow:

$$P_n = -\rho_n \varphi_n \quad \text{ёки} \quad \vec{V}_n = grad\,\varphi_n . \qquad (6)$$

Taking into account that the volumetric concentration of each phase in the mixture remains unchanged, and using the continuity equation together with equation (6), which indicates that the flow in each phase is potential, we substantiate the possibility of expressing the sought velocity potential φ_n through the pressure impulse [9].

As a result of an instantaneous impulse, a potential flow arises in the dispersed mixture, and due to the positive effect of the impulse, the distribution of velocities of the mixture phases is described by equation (6).

The instantaneous action of a pressure impulse on a two-layer flow is of a potential nature. Based on this property, studies [10–12] investigate the characteristics of standing waves on the free surface of the mixture and on the layered, nonhomogeneous interface. Here, the upper layer G_1 is assumed to have the parameters $\rho_{nc}^{(1)}$, $f_n^{(1)}$, and $\vec{V}_n^{(1)}$.

Taking into account that the oscillations are very small, the layer boundaries are considered to be free surfaces, and the velocity potentials in both layers are expressed in the form $\varphi_n^{(m)}(x, y, t)$.

Using the above considerations, we see that the velocity potentials introduced in equation (6) satisfy the Laplace equation through the continuity equation (3).

$$\nabla^2 \varphi_n^{(m)} = 0 . \qquad (7)$$

The solution of the Laplace equation has the following form:

$$\varphi_n^m(x, y, t) = \left[A_n^m e^{ky} + B_n^m e^{-ky}\right] \cos kx \, \cos \delta t . \qquad (8)$$

Here, the coefficients $A_n^{(m)}$ and $B_n^{(m)}$ are determined from the boundary conditions of the layers [13] and are written using the Lagrange–Cauchy integral.

To determine the above coefficients, the system of homogeneous linear algebraic equations presented in [14] is used. The solution of this system yields the relationship between the wave number k and the frequency of standing waves δ. Using this relationship, the wavelength and the period of the standing wave can be found as:

$$\lambda = \frac{2\pi}{k}, \quad \tau = \frac{2\pi}{\delta} .$$

Here, τ is the period of the standing wave. As the wavelength increases, the period of the standing waves also increases. In addition, by introducing the vector velocity, the relationship between the particle velocities of the mixture phases and the reduced and true densities [15] with the vector velocity is given by:

$$\vec{V}_{\tilde{m}}^{m} = \frac{\rho_1^{(m)} \vec{V}_1^{(m)}}{\rho^{(m)}} + \frac{\rho_2^{(m)} \vec{V}_2^{(m)}}{\rho^{(m)}} . \qquad (9)$$

Here, $\rho^{(m)} = \rho_1^{(m)} + \rho_2^{(m)}$, $\rho_n^{(m)} = \rho_{ni}^{(m)} f_n^{(m)}, \ f_1^{(m)} + f_2^{(m)} = 1.$

If the lower layer of the dispersed mixture flow is infinitely deep, i.e., $H \to \infty$, then the solution of the above equation for the wave number k and the wave frequency δ takes the following form:

$$\delta_1^2 = gk, \quad \delta_2^2 = gk \frac{th\, kH}{\hat{\rho} + th\, kH} .$$

The corresponding coefficients for each layer, as well as the expressions for the coefficients of the second layer written in terms of the coefficients of the first layer, take the following form [16]:

$$A^{II} = \left[1 + \frac{\delta^2 - gk}{\delta^2 + gk} e^{2kH}\right] \frac{A^I}{1 - e^{-2kH}},$$

$$B^{II} = \left[1 + \frac{\delta^2 - gk}{\delta^2 + gk} e^{2kH}\right] \frac{A^I - e^{-2kH}}{1 - e^{-2kH}}, \qquad (10)$$

$$B^I = \frac{\delta^2 - gk}{\delta^2 + gk} e^{2kH} A^I.$$

In the limit $H \to \infty$, let us determine the interfaces separating the layers and the vertical displacement functions of these interfaces, $\eta_1(x,t)$ and $\eta_2(x,t)$.

For the case where the lower layer of the dispersed mixture is infinitely deep ($H \to \infty$), we obtain the following equation for the wave parameters and the oscillation frequency:

$$\hat{\rho}(\delta^4 - g^2k^2) + (\delta^2 gk)^2(1 + thkH)\hat{\rho} -$$

$$-(\delta^4 - g^2k^2)thkH = 0.$$

By separating this equation into two equations, we determine the oscillation frequency in the form (6). In the first case, using equations (8) and (10), we determine the coefficients $A_n^{(m)}$ and $B_n^{(m)}$.

$$A^{II} = A^I, \quad B^I = 0, \quad B^{II} = 0.$$

In this case, the velocity potentials are determined by the following equations:

$$\varphi^{\mathrm{I}} = A^{\mathrm{I}} e^{ky} \cos kx \cos \delta t,$$

$$\varphi^{\mathrm{II}} = A^{\mathrm{I}} e^{ky} \cos kx \cos \delta t. \qquad (11)$$

In the second case, the oscillation amplitudes are equal to each other, while the oscillation amplitudes of the mixture free surface and of the interfaces separating the mixture layers differ from each other by a factor of e^{kH}. The propagation of wave velocities in both layers is identical and is expressed by the following function:

$$V^{I}_{c\grave{i}} = V^{I}_{c\grave{i}} = kA^{I} e^{ky} \cos \delta t.$$

The oscillatory elevations of the free surfaces of both phases and the interfaces separating them, as well as the boundaries and free surfaces formed under the action of the pressure impulse, are determined by the following equations:

$$\eta_1 = \frac{1}{g}\frac{\partial \varphi^I}{\partial t}\bigg|_{y=H} = 0,$$

$$\eta_2 = \frac{\delta}{g} A^I \cos kx \sin \delta t. \qquad (12)$$

As a result of oscillatory motion, the free surfaces of the perturbed layers are determined by the following formulas:

$$y = \eta_1(x, y, t),$$

$$y = H - \frac{\delta}{g} A^I \cos kx \sin \delta t. \qquad (13)$$

Now let us determine the displacement of the boundary line separating the two layers under the action of the pressure impulse [17]. The equation of the boundary line separating the two layers is given by:

$$y(x,t) = -\frac{A_1^I\delta}{g}e^{kH}\cos kx\sin\delta t. \tag{14}$$

From equations (10) and (12), it follows that as the thickness of the upper layer increases, the oscillation amplitudes at the free surface and at the interface separating the two layers also increase. A comparison of equations (12) and (14) shows that the oscillation amplitude of the lower layer exceeds that of the upper layer by a factor of e^{kH}.

Let us now consider the second relationship, namely, the dependence of the wave oscillation frequency on the wavelength and on the distribution of velocities in the upper layer. For this purpose, we write the expressions for the wave propagation velocity potential and its components along the coordinate axes:

$$\varphi^I(x,y,t) = A^I(e^{ky} + \Lambda_0 e^{2kh}e^{-ky})\cos\delta t\cos kx, \tag{15}$$

$$U^I(x,y,t) = \frac{\partial\varphi^I}{\partial x} = \left[A^I e^{ky} + \Lambda_0 e^{2kH}e^{-ky}\right](-k)\sin kx\cos\delta t,$$

$$V^I(x,y,t) = \frac{\partial\varphi^I}{\partial y} = kA^I\left[e^{ky} - \Lambda_0 e^{2kh}e^{-ky}\right](-k)\cos kx\cos\delta t$$

and we determine the vertical displacement function of the points on the interfaces separating the layers as follows:

$$\eta_1 = -\frac{\delta A^I}{g} = (1+\Lambda_0 e^{kH})\sin\delta t\cos kx. \tag{16}$$

From this equation, the common surface of wave propagation in both layers is expressed by the following relation:

$$y = H - \frac{\delta A^I}{g} = (1+\Lambda_0 e^{kH})\sin\delta t\cos kx. \tag{17}$$

We determine the deviation of the interface separating the flows from its equilibrium (pre-oscillation) position as follows:

$$\eta_2 = \frac{1}{g(\hat\rho-1)}\left(\hat\rho\frac{\partial\varphi^{II}}{\partial t} - \frac{\partial\varphi^I}{\partial t}\right)_{y=0}.$$

or

$$\eta_2 = -\frac{\delta A^I}{g(\hat\rho-1)}\left[(1-\Lambda_0 e^{kH})\hat\rho - (1+\Lambda_0 e^{-2kH})\right]\cos kx\sin\delta t. \tag{18}$$

Hence, the equation of the interface between the two layers can be written in the following form:

$$y = \frac{\delta A^I}{g(\hat\rho-1)}\left[1+\Lambda_0 e^{-2kH} - \hat\rho(1-\Lambda_0 e^{-2kH})\right]. \tag{19}$$

The propagation of velocities in the lower layer is determined by the following equations:

$$\left.\begin{aligned} U^{II} &= -kA^I(1-\Lambda_0 e^{-2kH})e^{ky}\cos k\delta\cos kx, \\ V^{II} &= kA^I(1-\Lambda_0 e^{-2kH})e^{ky}\cos k\delta\cos kx. \end{aligned}\right\} \tag{20}$$

Using equations (6), (10), and (16), we determine the displacement of the free surface of the upper layer from its equilibrium position.

Thus, for standing waves, all parameters of wave motion are obtained. Unlike single-phase fluids, in two-phase fluids the displacement generated by oscillations also reflects interphase interaction and mutual penetration of phases, which are taken into account in the equation of motion through the term $K(\vec{V}_p - \vec{V}_n)$ [18].

$$\tau_{xy} = \left(\mu_2^{(m)} f_2^{(m)} + \mu_1^{(m)} f_1^{m}\right) -$$

$$-k^2\left[\left(A_1^{(m)} + A_2^{(m)}\right)e^{ky} - \left(B_1^{(m)} - B_2^{(m)}\right)e^{-ky}\right]\sin kx \cos \delta t. \tag{21}$$

This expression consists of an invariant part (the first term) and a variable part that depends on the wave number.

CONCLUSIONS AND SUGGESTIONS

In this article, the propagation of small perturbations in a two-layer dispersed medium has been analyzed. Using mathematical models and hydrodynamic equations, it has been shown that the propagation radius of small perturbations and the impulse flow exhibit significant changes over short time intervals. The research results indicate that, in a two-phase medium, interphase interaction plays a crucial role in the propagation process, while propagation efficiency and impulse transmissibility depend on various parameters, in particular layer thickness, viscosity, and granulometric composition. At the same time, the results of modeling and calculations suggest that, in practice, the propagation process can be controlled by optimizing the above-mentioned factors.

In conclusion, it has been demonstrated that the propagation of small perturbations in a two-layer dispersed medium is a multifactorial process involving interphase interaction, turbulence, and viscosity. Modern hydrodynamic models in this field make it possible to study their underlying mechanisms in greater detail.

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