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CONTENTS

CONTENTS

MODELING MAGNETOELASTIC VIBRATIONS OF COMPLEX-SHAPED TRANSVERSELY ISOTROPIC PLATES USING THE R-FUNCTION METHOD

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Annotation: In this study, the Hamilton–Ostrogradsky variational principle is employed in conjunction with the Timoshenko hypothesis, while the Cauchy relations, Hooke's law, and Maxwell's electromagnetic tensor formulations are integrated to characterize the physico-mechanical properties of the deformation process. As a result, a general variational functional incorporating potential energy, kinetic energy, and the work performed by external forces is constructed. Based on this functional, a mathematical model in the form of a system of linear partial differential equations, supplemented by initial and boundary conditions imposed on the displacement functions, is derived. An efficient computational algorithm is developed for the proposed model, and numerical solutions and corresponding graphical results are obtained.

Keywords: Hamilton–Ostrogradsky principle, Timoshenko hypothesis, Cauchy relations, Hooke's law, Maxwell's tensor, deformation process, variational functional, partial differential equations, displacement functions, computational algorithm.

Annotatsiya: Ushbu maqolada Gamilton–Ostrogradskiy variatsion prinsipi Timoshenko gipotezasi bilan birgalikda qo'llanilib, deformatsiya jarayonining fizik-mexanik xususiyatlarini tavsiflash uchun Koshi munosabatlari, Guk qonuni va Maksvell elektromagnit tenzori ifodalari kompleks tarzda qo'llanildi. Natijada potensial energiya, kinetik energiya va tashqi kuchlar bajargan ishni o'z ichiga oluvchi umumiy variatsion funksional tuzildi. Mazkur funksional asosida siljish funksiyalariga qo'yilgan boshlang'ich va chegaraviy shartlar bilan to'ldirilgan chiziqli xususiy hosilali differensial tenglamalar tizimi ko'rinishidagi matematik model hosil qilindi. Taklif etilgan model uchun samarali hisoblash algoritmi ishlab chiqildi hamda sonli yechimlar va ularga mos grafik natijalar olindi.

Kalit so'zlar: Gamilton–Ostrogradskiy prinsipi, Timoshenko gipotezasi, Koshi munosabatlari, Guk qonuni, Maksvell tenzori, deformatsiya jarayoni, variatsion funksional, xususiy hosilali differensial tenglamalar, siljish funksiyalari, hisoblash algoritmi.

Аннотация: В данной статье вариационный принцип Гамильтона–Остроградского применяется совместно с гипотезой Тимошенко, а соотношения Коши, закон Гука и формулировки электромагнитного тензора Максвелла используются комплексно для описания физико-механических свойств процесса деформирования. В результате построен общий вариационный функционал, включающий потенциальную и кинетическую энергию, а также работу внешних сил. На основе данного функционала получена математическая модель в виде системы линейных дифференциальных уравнений в частных производных, дополненной начальными и граничными условиями, заданными для функций перемещений. Для предложенной модели разработан эффективный вычислительный алгоритм, получены численные решения и соответствующие графические результаты.

Ключевые слова: принцип Гамильтона–Остроградского, гипотеза Тимошенко, соотношения Коши, закон Гука, тензор Максвелла, процесс деформирования, вариационный функционал, дифференциальные уравнения в частных производных, функции перемещений, вычислительный алгоритм.

INTRODUCTION

The deformation and vibration characteristics of anisotropic composite plates under the influence of a magnetic field are examined. The effects of the magnetic field on stiffness and energy distribution are analyzed using analytical and numerical models, and general equations for transversely isotropic models are also presented. The results provide a detailed explanation of how magnetic forces affect plate stability and related dynamic processes [1].

The vibrations of transversely isotropic plates subjected to an electromagnetic field have also been investigated. The directional elastic properties of transversely isotropic materials are analyzed in relation to electromagnetic forces. Frequency, amplitude, and energy transfer are studied with respect to electromagnetic loading. These studies provide a fundamental basis for the magneto-elastic modeling of transversely isotropic plates [2].

The dynamic vibrations of anisotropic plates under magneto-elastic and thermal fields are considered. A generalized elasticity model applicable to transversely isotropic materials is presented, and the influence of magneto-thermal stresses on vibration behavior is thoroughly analyzed [3].

The free and forced vibrations of plates composed of transversely isotropic materials in a magnetic field have been studied. Magneto-elastic coupling, stiffness variation, and energy dissipation are modeled. Using the smart-plate approach, the directional characteristics of transversely isotropic materials are clearly demonstrated [4].

The vibration analysis of plates with complex geometries is carried out using special analytical functions to accurately describe boundary regions. Although transversely isotropic materials are not directly considered, the proposed method can be adapted for transversely isotropic plates. An approach closely related to the R-function method is available for analyzing complex geometries [5].

LITERATURE REVIEW

The coupling between elasticity and an electromagnetic field in anisotropic layered plates is investigated, and the interlayer directional properties of the material—particularly important for transversely isotropic models—are analyzed. The study demonstrates how a magnetic field influences vibration frequencies and plate deformation modes [6]. The combined effects of temperature, electric, and magnetic fields on anisotropic thin plates are also examined, and the complete set of coupling equations for transversely isotropic plates is presented. Their influence on stress and vibration states is mathematically modeled [7].

The modeling of magnetoelastic vibrations in anisotropic plates represents an important research area at the intersection of solid mechanics, structural dynamics, elasticity theory, and electromagnetic mechanics. This subject has attracted considerable attention because anisotropic and transversely isotropic plates are widely used in aerospace structures, composite engineering, microelectromechanical systems, magnetostrictive devices, and other advanced technological applications. Unlike isotropic plates, transversely isotropic materials exhibit direction-dependent mechanical properties, which significantly influence their natural frequencies, mode shapes, and vibration characteristics. The presence of a magnetic field further increases the complexity of the problem by introducing electromagnetic forces and magnetoelastic coupling into the governing equations.

Research methodology

A linear mathematical model of the plate is derived based on the Hamilton–Ostrogradsky variational principle [11]. In formulating the plate's equation of motion, the Timoshenko hypothesis is employed to describe the displacement field.

$$u_1 = u(x,y,t) - z\frac{\partial w}{\partial x} + \frac{J_0}{G'}\varphi(x,y,t),\ u_2 = v(x,y,t) - z\frac{\partial w}{\partial y} + \frac{J_0}{G'}\psi(x,y,t), u_3 = w(x,y,t). \quad (1)$$

Here: $u, v, w -$ Displacement: $\varphi, \psi -$ Angle of twist.

The general form of the Hamilton–Ostrogradsky variational principle is given in [2].

$$\delta\int_t (K - \Pi + A)dt = 0; \qquad (2)$$

Here, δ is the variation operator, K is the kinetic energy, Π is the potential energy, and A is the work performed by external body and surface forces.

In calculating the variation of the kinetic energy, the following relation is used. The general form of the variation of kinetic energy is given by [12]:

$$\int_t \delta \hat{E} dt = \int_t \int_V \left(\rho \frac{\partial u_1}{\partial t} \delta \frac{\partial u_1}{\partial t} + \rho \frac{\partial u_2}{\partial t} \delta \frac{\partial u_2}{\partial t} + \rho \frac{\partial u_3}{\partial t} \delta \frac{\partial u_3}{\partial t} \right) dV dt; \tag{3}$$

Here, ρ is the material density, u is the displacement, V is the volume, and t is time.

Integration by parts with respect to time is performed for the variation of the kinetic energy. As a result, the general expression for the variation of kinetic energy is obtained as follows [13]:

$$\begin{aligned} \int_t \delta K dt = \int_x \int_y \Big[& (\rho h \frac{\partial u}{\partial t} + \rho h \frac{J_0}{G'} \frac{\partial \varphi}{\partial t}) \delta u + (\rho h \frac{\partial v}{\partial t} + \rho h \frac{J_0}{G'} \frac{\partial \psi}{\partial t}) \delta v + \rho h \frac{\partial w}{\partial t} \delta w + (\rho h \frac{J_0}{G'} \frac{\partial u}{\partial t} + \rho h \frac{J_0^2}{G'^2} \frac{\partial \varphi}{\partial t}) \delta \varphi + \\ & + (\rho h \frac{J_0}{G'} \frac{\partial v}{\partial t} + \rho h \frac{J_0^2}{G'^2} \frac{\partial \psi}{\partial t}) \delta \psi \Big] dxdy \Big|_t - \int_t \int_x \int_y \Big[(\rho h \frac{\partial^2 u}{\partial t^2} + \rho h \frac{J_0}{G'} \frac{\partial^2 \varphi}{\partial t^2}) \delta u + (\rho h \frac{\partial^2 v}{\partial t^2} + \rho h \frac{J_0}{G'} \frac{\partial^2 \psi}{\partial t^2}) \delta v + (\rho h \frac{\partial^2 w}{\partial t^2} - \\ & - \rho \frac{h^3}{12} \frac{\partial^4 w}{\partial x^2 \partial t^2} - \rho \frac{h^3}{12} \frac{\partial^4 w}{\partial y^2 \partial t^2}) \delta w + (\rho h \frac{J_0}{G'} \frac{\partial^2 u}{\partial t^2} + \rho h \frac{J_0^2}{G'^2} \frac{\partial^2 \varphi}{\partial t^2}) \delta \varphi + (\rho h \frac{J_0}{G'} \frac{\partial^2 v}{\partial t^2} + \rho h \frac{J_0^2}{G'^2} \frac{\partial^2 \psi}{\partial t^2}) \delta \psi \Big] dxdydt \end{aligned} \tag{4}$$

(1) Based on the Hamilton–Ostrogradsky variational principle, we next determine the general form of the variation of potential energy.
To obtain the general expression for the variation of potential energy, the linear strain components are varied using the Cauchy relations and the Timoshenko hypothesis (2).
As a result, the general form of the variational potential energy given in (5) is obtained.
false

$$\begin{aligned} \delta \ddot{I} = & \int_y \left[N_x \delta u + \frac{1}{2} N_{xy} \delta v - M_x \delta \frac{\partial w}{\partial x} + \frac{\partial M_x}{\partial x} \delta w - M_{xy} \delta \frac{\partial w}{\partial y} + \frac{J_0}{G'} N_x \delta \varphi + \frac{1}{2} \frac{J_0}{G'} N_{xy} \delta \psi \right] \Bigg|_x dy + \\ & + \int_x \left[N_y \delta v + \frac{1}{2} N_{xy} \delta u + \frac{\partial M_{xy}}{\partial x} \delta w + \frac{\partial M_y}{\partial y} \delta w - M_y \delta \frac{\partial w}{\partial y} + \frac{1}{2} \frac{J_0}{G'} N_{xy} \delta \varphi + \frac{J_0}{G'} N_y \delta \psi \right] \Bigg|_y dx - \int_x \int_y \Big[(\frac{\partial N_x}{\partial x} + \frac{1}{2} \frac{\partial N_{xy}}{\partial y}) \delta u + \\ & + (\frac{\partial N_y}{\partial y} + \frac{1}{2} \frac{\partial N_{xy}}{\partial x}) \delta v + (\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2}) \delta w + (\frac{J_0}{G'} \frac{\partial N_x}{\partial x} + \frac{1}{2} \frac{J_0}{G'} \frac{\partial N_{xy}}{\partial y}) \delta \varphi + (\frac{J_0}{G'} \frac{\partial N_y}{\partial y} + \frac{1}{2} \frac{J_0}{G'} \frac{\partial N_{xy}}{\partial x}) \delta \psi \Big] dxdy \end{aligned} \tag{5}$$

Here: M_x, M_y, M_{xy} bending and twisting moments; N_x, N_y, N_{xy} normal and shear forces.

Taking the electromagnetic forces [14] into account, and appropriately rearranging the terms in accordance with the variation of the work performed by external forces, we carry out the necessary operations—namely, integration, integration by parts, and simplification of similar terms. As a result, the expression given in (6) is obtained.

$$\begin{aligned} \int_t \delta A dt = \int_t \int_x \int_y \Big[& (\xi_x + R_x + q_x + T_{zx}) \delta u + \frac{J_0}{G'} (\xi_x + R_x + q_x + T_{zx}) \delta \varphi + (\xi_y + R_y + q_y + T_{zy}) \delta v + \\ & + \frac{J_0}{G'} (\xi_y + R_y + q_y + T_{zy}) \delta \psi + (\xi_z + R_z + q_z + T_{zz}) \delta w \Big] dxdydt - \int_t \int_x \int_y \left[z(q_x + T_{zx}) \delta \frac{\partial w}{\partial x} + z(q_y + T_{zy}) \delta \frac{\partial w}{\partial y} \right] \Bigg|_z dxdydt + \\ & + \int_t \int_y \left[(N_{Px} + N_{Txx}) \delta u + \frac{J_0}{G'} (N_{Px} + N_{Txx}) \delta \varphi + (N_{Py} + N_{Txy}) \delta v + \frac{J_0}{G'} (N_{Py} + N_{Txy}) \delta \psi + (N_{Pz} + N_{Txz}) \delta w \right] \Bigg|_x dydt + \\ & + \int_t \int_x \left[(N_{Fx} + N_{Tyx}) \delta u + \frac{J_0}{G'} (N_{Fx} + N_{Tyx}) \delta \varphi + (N_{Fy} + N_{Tyy}) \delta v + \frac{J_0}{G'} (N_{Fy} + N_{Tyy}) \delta \psi + (N_{Fz} + N_{Tyz}) \delta w \right] \Bigg|_y dxdt \end{aligned} \tag{6}$$

Here $X, Y, Z, \rho K_x, \rho K_y, \rho K_z$ – Resulting body forces, $q_x = q_x^+ + q_x^-$, $q_y = q_y^+ + q_y^-$, $q_z = q_z^+ + q_z^-$, $P_x, P_y, P_z, T_{xx}, T_{xy}, T_{xz}, F_x, F_y, F_z, T_{yx}, T_{yy}, T_{yz}$ – Resulting contour forces. $\xi_{zx} = \xi_{zx}^+ + \xi_{zx}^-$, $\xi_{zy} = \xi_{zy}^+ + \xi_{zy}^-$,

$\xi_{zz} = \xi_{zz}^{+} + \xi_{zz}^{-}$ –Resulting surface forces

In computing the electromagnetic forces, an expression based on Maxwell's electromagnetic tensor is employed. This approach enables the accurate determination of the electromagnetic forces generated in a thin plate under the influence of an external field [15].

$$R = \rho K = \frac{1}{4\pi}(\mathrm{rot}(\mathrm{rot}(\mathrm{U\,x\,H})))\,\mathrm{x\,H}. \qquad (7)$$

here: $U(u_1, u_2, u_3)$ – Displacement vector; $H(H_x, H_y, H_z)$ – Magnetic field strength vector.

Maxwell's electrodynamic stress tensor is a fundamental physical quantity that describes the mechanical influence of an electromagnetic field on a material body. It makes it possible to determine the energy and momentum densities of the electromagnetic field, as well as the forces and moments transmitted by the field to the body. Consequently, the Maxwell stress tensor plays a key role in determining the distribution of surface forces arising on the plate under electromagnetic loading [16].

$$\begin{cases} \xi_{ik} = \dfrac{1}{4\pi}\left[H_i h_k + h_i H_k\right] - \dfrac{\acute{a}_{ik}}{4\pi}\vec{h}\vec{H}, \\ \xi_{ik}^{e} = \dfrac{1}{4\mu\pi}\left[H_i^e h_k^e + h_i^e H_k^e\right] - \dfrac{\acute{a}_{ik}}{4\pi}\vec{h}^e\vec{H}^e. \end{cases} \quad here \quad {}_{ik} = \begin{cases} 0, i \neq k, \\ 1, i = k. \end{cases} \qquad (8)$$

Based on (8), the surface forces and contour (boundary) forces are expressed accordingly.

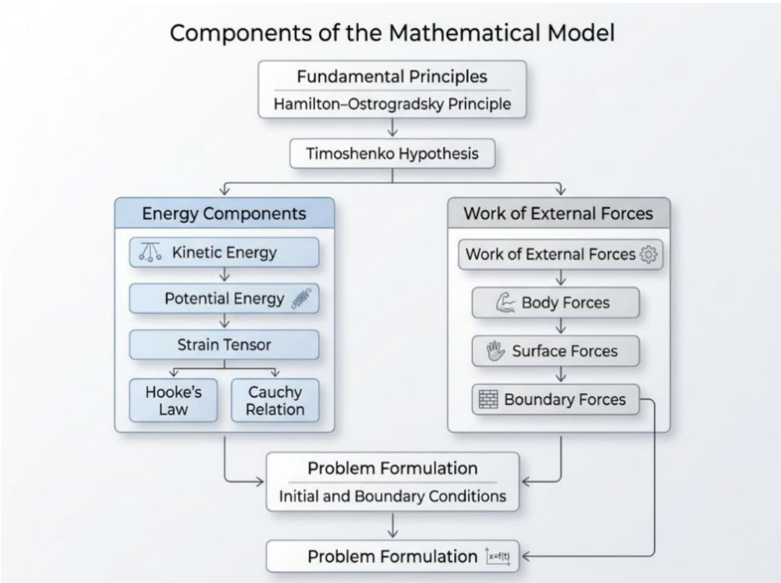

Figure 1. *Structure of the mathematical model based on the Hamilton–Ostrogradsky principle and the Timoshenko hypothesis.*

The variations of the potential energy, kinetic energy, and work obtained for the magneto-elastic vibration model of the transversely isotropic plate are substituted into equation (2), and the general equation of motion is derived. The corresponding initial and boundary conditions are also presented.

$$\begin{cases} \rho h\frac{\partial^2 u}{\partial t^2}+\rho h\frac{J_0}{G'}\frac{\partial^2 \varphi}{\partial t^2}+\frac{\partial N_x}{\partial x}+\frac{1}{2}\frac{\partial N_{xy}}{\partial y}+\xi_x+R_x+q_x+T_{zx}=0 \\ \rho h\frac{\partial^2 v}{\partial t^2}+\rho h\frac{J_0}{G'}\frac{\partial^2 \psi}{\partial t^2}+\frac{\partial N_y}{\partial y}+\frac{1}{2}\frac{\partial N_{xy}}{\partial x}+\xi_y+R_y+q_y+T_{zy}=0 \\ \rho h\frac{\partial^2 w}{\partial t^2}-\rho\frac{h^3}{12}\frac{\partial^4 w}{\partial x^2\partial t^2}-\rho\frac{h^3}{12}\frac{\partial^4 w}{\partial y^2\partial t^2}+\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_y}{\partial y^2}+\xi_z+R_z+q_z+T_{zz}=0 \\ \rho h\frac{J_0}{G'}\frac{\partial^2 u}{\partial t^2}+\rho h\frac{J_0^2}{G'^2}\frac{\partial^2 \varphi}{\partial t^2}+\frac{J_0}{G'}\frac{\partial N_x}{\partial x}+\frac{1}{2}\frac{J_0}{G'}\frac{\partial N_{xy}}{\partial y}+\frac{J_0}{G'}(\xi_x+R_x+q_x+T_{zx})=0 \\ \rho h\frac{J_0}{G'}\frac{\partial^2 v}{\partial t^2}+\rho h\frac{J_0^2}{G'^2}\frac{\partial^2 \psi}{\partial t^2}+\frac{J_0}{G'}\frac{\partial N_y}{\partial y}+\frac{1}{2}\frac{J_0}{G'}\frac{\partial N_{xy}}{\partial x}+\frac{J_0}{G'}(\xi_y+R_y+q_y+T_{zy})=0. \end{cases} \quad (9)$$

Initial condition:

$$\left.\left(\rho h\frac{\partial u}{\partial t}+\rho h\frac{J_0}{G'}\frac{\partial \varphi}{\partial t}\right)\right|_t=0, \qquad \left.\left(\rho h\frac{\partial v}{\partial t}+\rho h\frac{J_0}{G'}\frac{\partial \psi}{\partial t}\right)\right|_t=0,$$

$$\left.\left(\rho h\frac{\partial w}{\partial t}-\rho\frac{h^3}{12}\frac{\partial^3 w}{\partial x^2\partial t}-\rho\frac{h^3}{12}\frac{\partial^3 w}{\partial y^2\partial t}\right)\right|_t=0, \left.\left.\rho\frac{h^3}{12}\frac{\partial^2 w}{\partial t\partial x}\delta w\right|_x\right|_t=0,\ \left.\left.\rho\frac{h^3}{12}\frac{\partial^2 w}{\partial t\partial y}\delta w\right|_y\right|_t=0; \quad (10)$$

Boundary condition:

$$\begin{cases} N_x\delta u\big|_x=0,\ \frac{1}{2}N_{xy}\delta v\Big|_x=0,\ -M_x\delta\frac{\partial w}{\partial x}\Big|_x=0,\ -M_{xy}\delta\frac{\partial w}{\partial y}\Big|_x=0, \frac{J_0}{G'}N_x\delta\varphi\Big|_x=0, \frac{1}{2}\frac{J_0}{G'}N_{xy}\delta\psi\Big|_x=0. \\ N_y\delta v\big|_y=0,\ \frac{1}{2}N_{xy}\delta u\Big|_y=0,\ -M_y\delta\frac{\partial w}{\partial y}\Big|_y=0,\ \frac{\partial M_{xy}}{\partial x}\delta w\Big|_y=0, \frac{\partial M_y}{\partial y}\delta w\Big|_y=0, \\ \frac{1}{2}\frac{J_0}{G'}N_{xy}\delta\varphi\Big|_y=0, \frac{J_0}{G'}N_y\delta\psi\Big|_y=0, (N_{Px}+N_{Txx})\delta u\big|_x=0, (N_{Py}+N_{Txy})\delta v\big|_x=0, \\ (N_{Pz}+N_{Txz})\delta w\big|_x=0, \frac{J_0}{G'}(N_{Px}+N_{Txx})\delta\varphi\Big|_x=0, \frac{J_0}{G'}(N_{Py}+N_{Txy})\delta\psi\Big|_x=0, \\ (N_{Fx}+N_{Tyx})\delta u\big|_y=0, (N_{Fy}+N_{Tyy})\delta v\big|_y=0, (N_{Fz}+N_{Tyz})\delta w\big|_y=0, \\ \frac{J_0}{G'}(N_{Fx}+N_{Tyx})\delta\varphi\Big|_y=0, \frac{J_0}{G'}(N_{Fy}+N_{Tyy})\delta\psi\Big|_y=0. \end{cases} \quad (11)$$

Here $u, v, w-$ Plate deflections; $h-$ Plate thickness; $M_{xx}, M_{yy}, M_{xy}-$ Bending and torsional moments; $N_{xx}, N_{yy}, N_{xy}-$ Normal and shear forces, $R_x, R_y, R_z, \xi_x, \xi_y, \xi_z-$ Electromagnetic field and body forces; $\xi_{zx}, \xi_{zy}, \xi_{zz}, q_x, q_y, q_z-$ Components of surface forces; $N_{Px}, N_{Py}, N_{Pz}, N_{Fx}, N_{Fy}, N_{Fz}, N_{Txx}, N_{Txy}, N_{Txz}, N_{Tyx}, N_{Tyy}, N_{Tyz}-$ Components of contour forces.

ANALYSIS AND RESULTS

A numerical analysis algorithm was developed for magneto-elastic plates with complex structural geometries by integrating the analytical R-function method (RFM), the Newmark method, the variational Bubnov–Galerkin approach, and modern computational techniques. The proposed algorithm enables a step-by-step solution of the problem and includes the following key procedures [17].

First, when analyzing domains with complex geometries, it is essential to construct their boundary equations with high accuracy. Modern technological processes require a high degree of precision in structural optimization, numerical analysis, and design workflows. Therefore, automating problems related to assessing the strength, long-term reliability, and stability of plates represents an important research task. These aspects depend not only on the physical parameters of the material but also directly on the geometric and structural characteristics of the domain under investigation [18].

The solution process consists of the following stages:

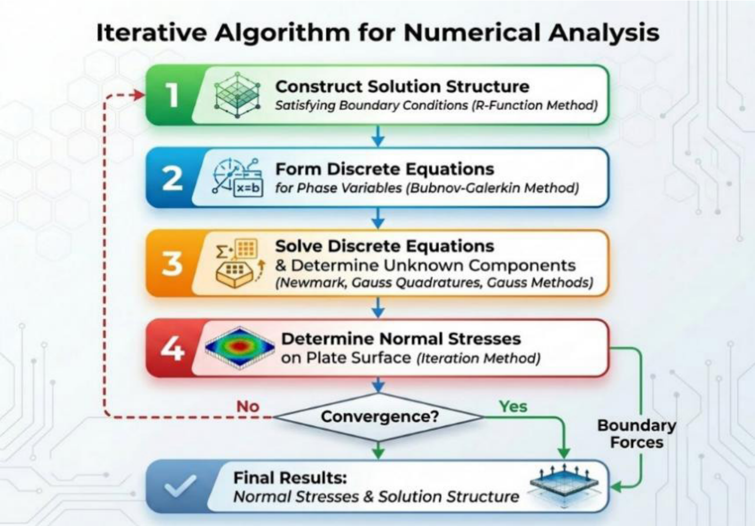

Figure 2. Iterative algorithm for modeling magnetoelastic vibrations of complex-shaped transversely isotropic plates using the R-function method.

The main essence of the R-function method: $S_2(x) \equiv (x \geq 0), x \in R^1, \;\; x \geq 0$ Let it be a predicate that takes the value **1 when** $x \geq 0$ **holds, and 0 when** $x \geq 0$ **holds otherwise.**

here: $x = (x_1, x_2, ..., x_n);\;\; S_2(x) = \{S_2(x_1), S_2(x_2), ..., S_2(x_n)\}$. Then the function $y = f(x)$ is called an **R-function** if there exists a logical function $Y = F(X)$, $X = (X_1, X_2, ..., X_n)$ such that the following condition is satisfied:

$$S_2[f(x)] = F[S_2(x)].$$

Here, the function $Y = F(X)$ is a logical function provided that the set X_i va B_2 ={0;1} contains only two elements, namely 0 and 1[22].

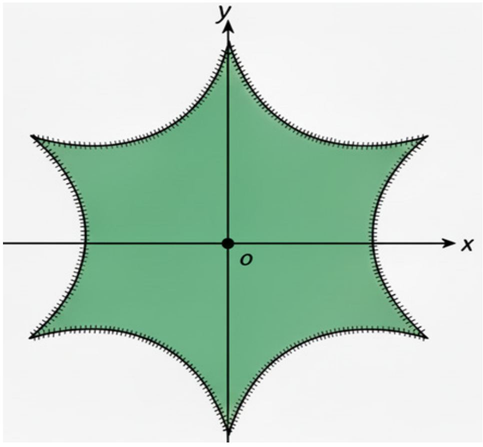

Figure 3. A complex shape with a sixsided rounded contour is represented with high precision by means of the Rfunction.

For this problem, the structural formulas corresponding to the primary (essential) boundary conditions are written as follows:

$u=\omega\Phi_1,\quad v=\omega\Phi_2,\quad w=\omega\Phi_3, \varphi=\omega\Phi_4,\quad \psi=\omega\Phi_5.$

$a=1,\ b=2,\ j=0.5, l=0.5, c=0.6, b_2=0.6, t=1s, H_x=H_y=H_z=15kE, Q_1=Q_2=Q_3=12;$
$E=118GPa, \nu=0.3, E_1=342.09GPa, E_2=357.73GPa, \nu_1=0.2586, \nu_2=0.2792, h=1sm, n=1, m=1.$
$E/G=2,6, C_1=C_2=C_3=10;$

here $a,\ j,l,b,\ m,n,c-$ Geometric parameters, $t=1s, H_x, H_y, H_z, Q_1, Q_2, Q_3-$ Mechanical parameters, $E,\nu-$ Young's modulus and Poisson's ratio for copper, E_1, E_2, ν_1, ν_2-

Computational Experiment–1: In this study, the deformation behavior of a transversely isotropic plate with rigidly fixed boundaries under the influence of electromagnetic field forces was investigated. Numerical results for the displacement function $w(x,y,t)$ were obtained along the Ox, Oy and Oz axes for the values given in $x\in[-1;1], y=0, t=1$. To comprehensively analyze the geometrically linear deformation state of thin transversely isotropic plates with complex structural shapes under the influence of electromagnetic field forces, graphical results were presented and analyzed to provide a clear understanding of the deformation process (Table 1).

Graphical results of the deformation state of transversely isotropic plates along the Ox, Oy false and Oz falseaxes.

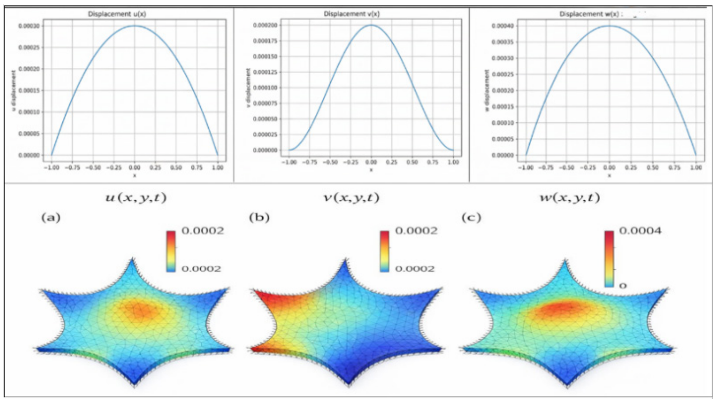

Figure-4. *This graphical representation shows the bending of the system along the Oz false axis when viewed from the directions of axes Ox false and Oy false.*

Computational experiment 2: In this study, the changes in twisting angles and deformation states of a plate with rigidly clamped boundaries, made of a transversely isotropic material, were investigated under the action of electromagnetic field forces. Numerical results of the displacement function $w(x,y,t)$ along the Oz axis at $x\in[-1;1], y=0, t=1$ values are presented, and graphical results are provided and analyzed to give a complete understanding of the deformation state (Table 2).

Graphical results of the twisting angles and deformation state of transversely isotropic plates along the Oyz, Oxz and Oz axes.

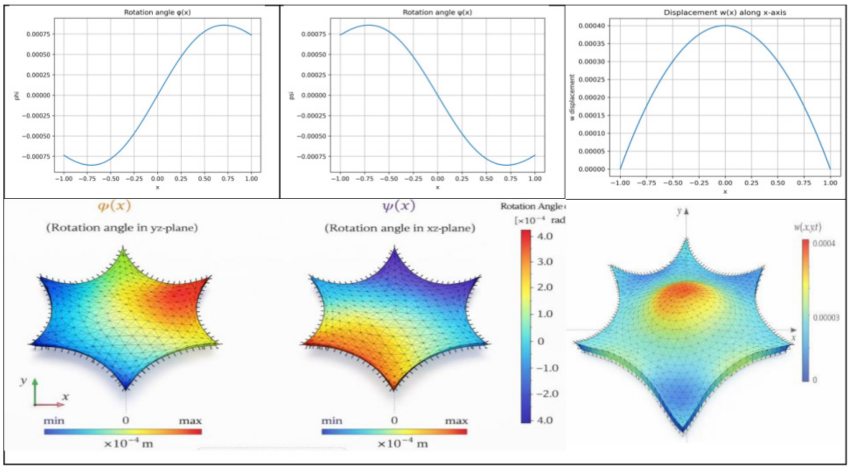

Figure-5. This graphical representation shows the bending of the plate along the Oz *false axis when viewed in projection onto the* Oyz, Oxz *axes.*

The numerical experiment shows that for a thin, complexshaped anisotropic transversely isotropic plate, the difference in bending when twisting angles are applied and when they are not can reach up to 19.3 percent.

CONCLUSION AND SUGGESTIONS

In this study, a mathematical model describing the electro-magneto-elastic state of linear transversely isotropic thin plates with complex geometrical shapes subjected to an electromagnetic field was developed. The influence of the electromagnetic field on the geometrically linear deformation of the plate was examined in detail. The model was constructed based on the Hamilton–Ostrogradsky variational principle using the Timoshenko hypothesis, the Cauchy relations, Hooke's law, and Maxwell's electromagnetic tensor formulations. On the basis of these theoretical principles, the variational forms of the potential energy, kinetic energy, and work performed by external forces were obtained.

As a result, a mathematical model in the form of a system of linear partial differential equations, supplemented by initial and boundary conditions for the displacement functions, was derived. A computational algorithm was developed based on the proposed model, and numerical methods were employed to determine the plate displacements. The numerical results were presented graphically and demonstrated that the electromagnetic field has a significant influence on the deformation response of a transversely isotropic plate.

The developed mathematical model provides a reliable theoretical and practical basis for analyzing complex-shaped transversely isotropic plates operating in an electromagnetic field, evaluating their stress–strain state, and supporting the design and analysis of electromechanical systems.

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